we may take $I_{\nu_0}^{(+)} + I_{\nu_0}^{(-)} \gg I^{\text{sat}}$, for from (5.2.5) and (5.3.3),

$$\frac{I_{\nu_0}^{(+)} + I_{\nu_0}^{(-)}}{I^{\text{sat}}} = \frac{g_0(\nu_0)}{g_t} - 1 = \frac{g_0(\nu_0)}{\sqrt{g_0(\nu_0)s/2l}} - 1 = \sqrt{\frac{2g_0(\nu_0)l}{s}} - 1 \gg 1. \tag{5.3.12}$$

In this case the laser transition is strongly saturated, that is, $\bar{N}_1 \approx \bar{N}_2 \approx \frac{1}{2}N_T$, and therefore

$$e_{\max} \approx \frac{P - \Gamma_{21}}{P}\frac{\nu_0}{\nu_{31}}. \tag{5.3.13}$$

Finally, we assume $P \gg \Gamma_{21}$ in order to have a large small-signal gain [Eq. (4.12.4)]. Thus

$$e_{\max} \approx \frac{\nu_0}{\nu_{31}} = \frac{E_2 - E_1}{E_3 - E_1}. \tag{5.3.14}$$

This is the theoretical upper limit to the power conversion efficiency. It is just the ratio of the quantum of energy $h\nu_0$ associated with the laser transition to the quantum of energy $h\nu_{31}$ associated with the pump transition of the three-level laser (Fig. 4.6). This ratio is called the *quantum efficiency* of the three-level laser. It is a property only of the energy-level structure of the active atoms. Similarly ν_0/ν_{30} is the quantum efficiency of the ideal four-level laser. In ruby and Nd : YAG lasers the pump level 3 is not a single, sharply defined level. Viewing them as approximately three- and four-level lasers, and using the numbers given in Section 4.10, we calculate that ruby and Nd : YAG lasers have quantum efficiencies $\leq$80% and $\leq$50%, respectively (Problem 5.4).

Needless to say, the quantum efficiency is seldom approached in real lasers. First of all, the input-to-output power conversion efficiency, of which the quantum efficiency is the theoretical upper limit, does not give the actual overall efficiency of operation of the laser. It only gives the fraction of the power *actually delivered* to the active medium that is converted to laser output power. There is no account of the efficiency with which the pump power is generated and delivered.

In a carefully designed cw ruby laser, for example, about 25% of the electric power used by the lamp is actually converted to radiation with frequencies lying within the pump bands of the chromium ion, and, of course, not all of this radiation is actually incident on the ruby rod. The fraction of the incident radiation actually absorbed by the ruby is about 4%, and of this only the fraction equal to the quantum efficiency may be used for lasing. All things considered, the actual operating efficiency of a cw ruby laser system is on the order of a tenth of a percent. Although much higher efficiencies are available with modern lasers, the point is that the quantum efficiency defined by (5.3.14) usually has little bearing on the actual operating efficiency of the complete laser system consisting of the pump, the gain cell, and the laser resonator.

5.4 EFFECT OF SPATIAL HOLE BURNING

The effect of spatial hole burning is to reduce the output intensity. This can be understood as follows.

The gain saturates according to the formula (4.13.9), that is,

$$g(\nu) = \frac{g_0(\nu)}{1 + 4(I_\nu^{(+)}/I_\nu^{\text{sat}})\sin^2 kz}, \tag{5.4.1}$$

where we have used Eq. (5.2.8) to write

$$I_\nu = I_\nu^{(+)} + I_\nu^{(-)} = 2I_\nu^{(+)}. \tag{5.4.2}$$

Our result (5.2.11) for the laser output intensity is based on the approximation of replacing $\sin^2 kz$ by its average value, that is, by ignoring the spatial dependence of the gain arising from the interference of the two traveling waves. We will now consider the effect of retaining the spatial variation (5.4.1) of the gain coefficient. In other words, we will now improve upon the uniform-field approximation by including the effect of spatial hole burning.

Equation (5.3.7) was written in the uniform-field approximation. Without this approximation we arrive at the expression

$$-h\nu\left(\frac{dN_2}{dt}\right)_{\text{stimulated emission}} = 2g(\nu)I_\nu \sin^2 kz = \frac{2g_0(\nu)I_\nu \sin^2 kz}{1 + 2(I_\nu/I_\nu^{\text{sat}})\sin^2 kz}. \tag{5.4.3}$$

This is the power (at frequency ν) per unit volume, *at the point* z, extracted from the gain medium by stimulated emission. Equation (5.3.7) follows when $\sin^2 kz$ is replaced by $\frac{1}{2}$, its average value over distances large compared with a wavelength.

The gain "clamping" condition (5.2.3) does not apply in the "exact" theory in which the gain and intensity vary with z. In other words, if g is a function of z we can no longer say that the gain and loss coefficients *at every point* in the gain medium are equal in steady-state oscillation. It must still be true, however, that the rate at which the field gains energy equals the rate at which it loses energy. The former follows from the generalization (5.4.3) of (5.3.7):

$$\int_0^l gI\,dz = 2g_0 I \int_0^l \frac{dz\,\sin^2 kz}{1 + 2(I/I^{\text{sat}})\sin^2 kz}, \tag{5.4.4}$$

where we have dropped subscript ν's to simplify the notation.

The rate of field intensity loss from the cavity is just

$$(t + s)I^{(+)} = \tfrac{1}{2}(t + s)I = g_t lI. \tag{5.4.5}$$

Note that the one-way intensity $I_\nu^{(+)} = I/2$ in the direction of the output mirror is independent of z. The right-hand sides of (5.4.4) and (5.4.5) must be equal in cw oscillation, and this equality determines I. From a table of integrals we find that, for $kl \gg 1$,

$$\int_0^l \frac{dz\,\sin^2 kz}{1 + 2(I/I^{\text{sat}})\sin^2 kz} \cong \frac{l}{2}\frac{I^{\text{sat}}}{I}\left(1 - \frac{1}{\sqrt{1 + 2I/I^{\text{sat}}}}\right), \tag{5.4.6}$$

and, therefore, from the equality of (5.4.4) and (5.4.5),

$$1 - \frac{1}{\sqrt{1 + 2I/I^{\text{sat}}}} = \frac{g_t I}{g_0 I^{\text{sat}}}. \tag{5.4.7}$$

This expression can be written more simply:

$$\sqrt{x} = \frac{2g_0}{g_t} - x, \tag{5.4.8}$$

where

$$x = 1 + \frac{2I}{I^{\text{sat}}}. \tag{5.4.9}$$

Squaring both sides of (5.4.8), we obtain a quadratic equation for x, with the two solutions

$$x = 1 + \frac{2I}{I^{\text{sat}}} = \frac{2g_0}{g_t} + \frac{1}{2} \pm \sqrt{\frac{2g_0}{g_t} + \frac{1}{4}}. \tag{5.4.10}$$

Since x should be equal to 1 ($I = 0$) when $g_0/g_t = 1$, the desired solution is the one with the minus sign in the last term on the right:

$$1 + \frac{2I}{I^{\text{sat}}} = \frac{2g_0}{g_t} + \frac{1}{2} - \sqrt{\frac{2g_0}{g_t} + \frac{1}{4}}, \tag{5.4.11}$$

or

$$I = I^{\text{sat}}\left(\frac{g_0}{g_t} - \frac{1}{4} - \sqrt{\frac{g_0}{2g_t} + \frac{1}{16}}\right). \tag{5.4.12}$$

The output intensity is $I^{\text{out}} = tI^{(+)} = (t/2)I$, exactly as in the uniform-field approximation in which spatial hole burning is not included. This is because I^{out} is determined directly by the *one-way* intensity $I^{(+)}$ (Fig. 5.1), and there are no interference terms to worry about. Thus

$$I^{\text{out}} = \frac{t}{2} I^{\text{sat}}\left(\frac{g_0(\nu)}{g_t} - \frac{1}{4} - \sqrt{\frac{g_0(\nu)}{2g_t} + \frac{1}{16}}\right), \tag{5.4.13}$$

which is different from the result (5.2.9) obtained when spatial hole burning is neglected.

Figure 5.2 shows the curve of output intensity vs. output coupling predicted by (5.2.11) for the example $g_0 l = 0.10$ and $s = 0.034$. Also shown is the curve predicted by the formula (5.4.13). The two predictions are seen to differ significantly, typically by about 30%. Thus, the effect of spatial hole burning is to *reduce* the output intensity, as already mentioned.

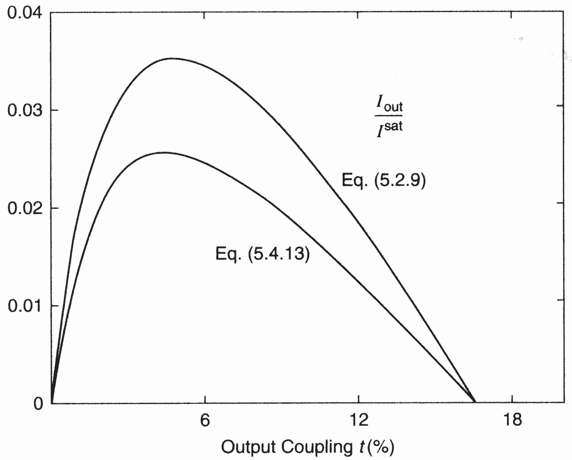

Figure 5.2 Effect of spatial hole burning on output intensity, assuming $g_0 l = 0.10$ and $s = 0.034$.

5.5 LARGE OUTPUT COUPLING

Our analysis of output power thus far has assumed that the output coupling is small and that the two traveling waves have equal intensities, $I^{(+)} = I^{(-)}$. We have also assumed that the time-averaged intensities $I^{(+)}$ and $I^{(-)}$ are independent of the axial coordinate z. We will now allow arbitrary output coupling and, therefore, allow the possibility that $I^{(+)}$ and $I^{(-)}$ may vary with z. We assume, however, that the variation of interest is much more gradual than the $\sin^2 kz$ variation due to spatial hole burning, and replace $\sin^2 kz$ by its average value $\frac{1}{2}$.

Thus, we work with the gain–saturation formula (5.2.4), which we now write in the form

$$g(z) = \frac{g_0}{1 + [I^{(+)}(z) + I^{(-)}(z)]/I^{\text{sat}}}. \tag{5.5.1}$$

For notational simplicity we have again suppressed the ν dependence of the various terms in this equation, but we indicate explicitly the z dependence. In principle, g_0 could also depend on z, for example, if the pumping rate P depends on z, but here we assume that it does not. In steady-state oscillation the intensities $I^{(+)}$ and $I^{(-)}$ satisfy Eqs. (4.3.4):

$$\frac{dI^{(+)}}{dz} = g(z)I^{(+)}(z), \tag{5.5.2a}$$

$$\frac{dI^{(-)}}{dz} = -g(z)I^{(-)}(z). \tag{5.5.2b}$$

We will assume that all cavity loss processes (output coupling, scattering, absorption) occur at the mirrors. Thus, we will not include terms accounting for "distributed" loss within the cavity.